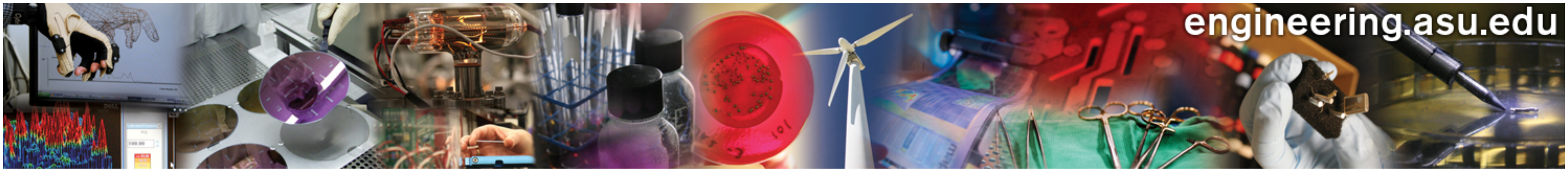

IRA A. FULTON SCHOOLS OF ENGINEERING

Leading engineering discovery and innovative education for global impact on quality of life.

Introduction to the Design and Analysis of Experiments

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Complex Engineered Systems

- What makes an engineered system complex?
  - Its large size.
  - Humans control their structure, operation, and evolution over time.
- Examples of complex engineering networks:
  - The power grid.
  - Transportation networks.
  - Computer networks (e.g., the Internet, etc.).

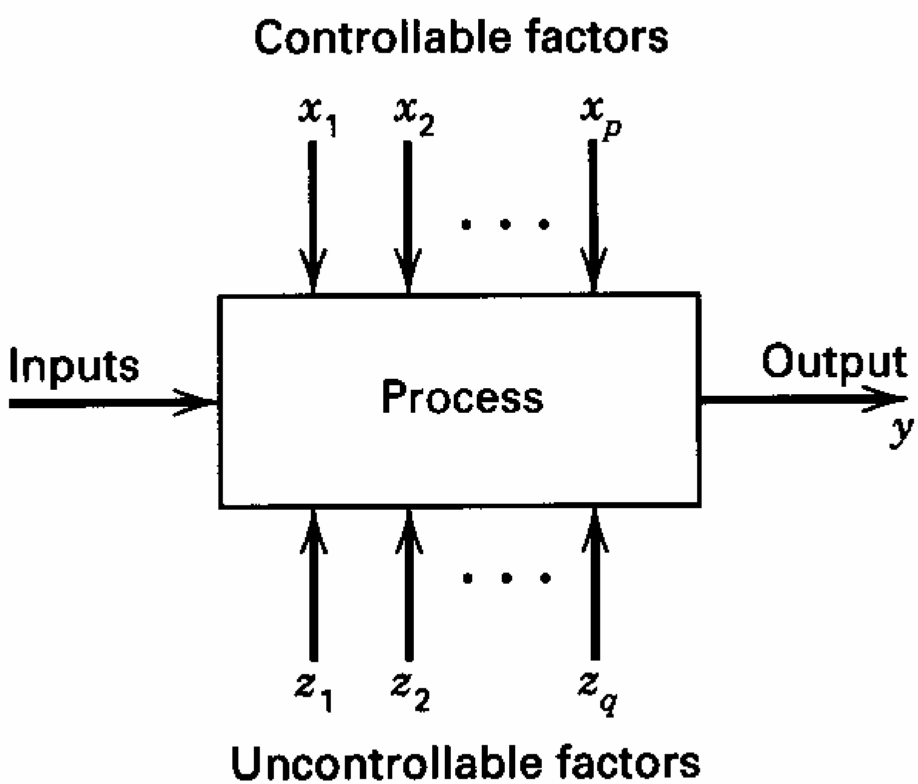
Experimentation

- Experimentation is a way to improve our understanding of CESs.
- Experiments are used widely for, e.g.:
  - Process characterization and optimization.
  - Improve the reliability and performance of products and processes.
  - Product/process design and development.
  - Achieve product and process robustness.

"All experiments are designed experiments;
some are poorly designed,
some are well-designed."

George E. P. Box

A General Model of a Process/System†



† "Design and Analysis of Experiments," by Douglas C. Montgomery, Wiley, 8th edition, 2013.

Four Eras in the History of DoE

- The agricultural origins, 1918-1940s:
  - R.A. Fisher and his co-workers.
  - Profound impact on agricultural science.
  - Factorial designs, ANOVA.
- The first industrial era, 1951- late 1970s:
  - Box and Wilson, response surfaces.
  - Applications in the chemical and process industries.

Four Eras in the History of DoE (cont'd)

- The second industrial era, late 1970s - 1990:
  - Quality improvement initiatives in many companies.
  - Taguchi and robust parameter design, process robustness.
- The modern era, beginning circa 1990.

Experimentation in CENs

- From a recent workshop report†:

  "The science of experiment design is widely used in science and engineering disciplines, but is often ignored in the study of complex engineered networks. This in turn has led to a shortage of simulations that we can believe in, of experiments driven by empirical data, and of results that are statistically illuminating and reproducible in this field."

† Networking and Information Technology Research and Development (NITRD), Large Scale Networking (LSN), Workshop Report on Complex Engineered Networks, September 2012.

Factorial Designs

- In a factorial experiment, all possible combinations of factor levels are tested.
- The golf experiment:
  - Type of driver.
  - Type of ball.
  - Walking versus riding.
  - Type of beverage.
  - Time of round, etc.

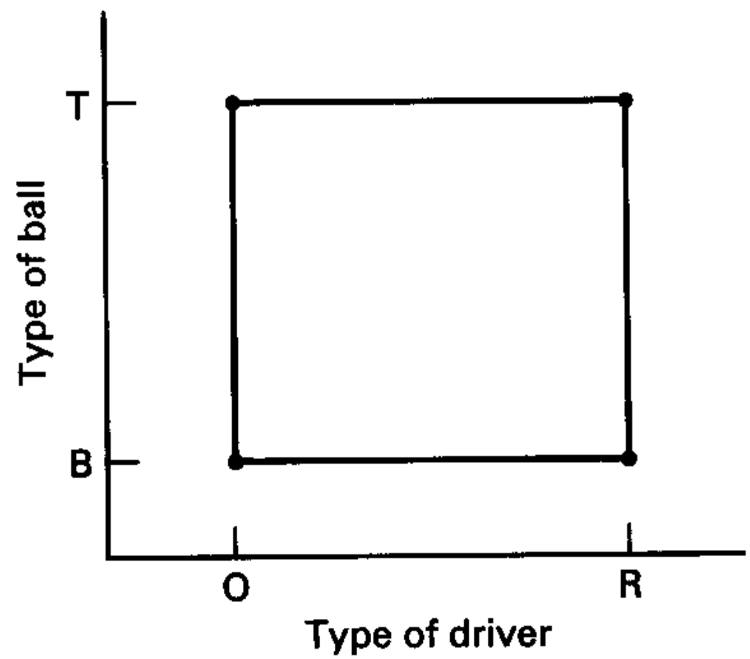


Figure 1-4 A two-factor factorial experiment involving type of driver and type of ball.

The Experimental Design

- An experiment is given by an N × k array.
  - The k columns correspond to the factors.
    - Each factor F_i, $1 \le i \le k$ has a set of levels L_i.
- Each of the N rows corresponds to a test in which each factor F_i is set to a level in L_i.
- For the two-factor factorial experiment:

	Ball	Driver
1	B	O
2	B	R
3	T	O
4	T	R

Factorial Designs with Several Factors

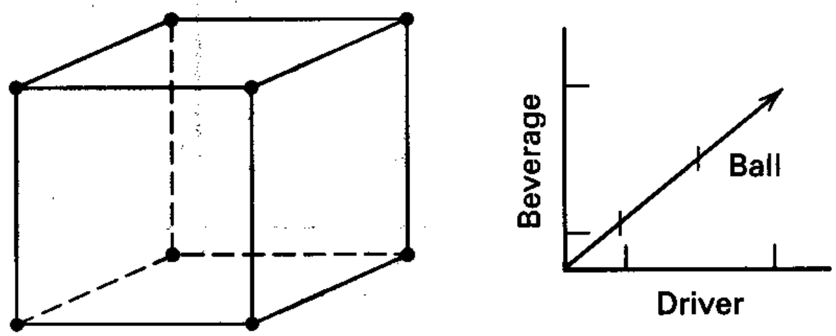


Figure 1-6 A three-factor factorial experiment involving type of driver, type of ball, and type of beverage.

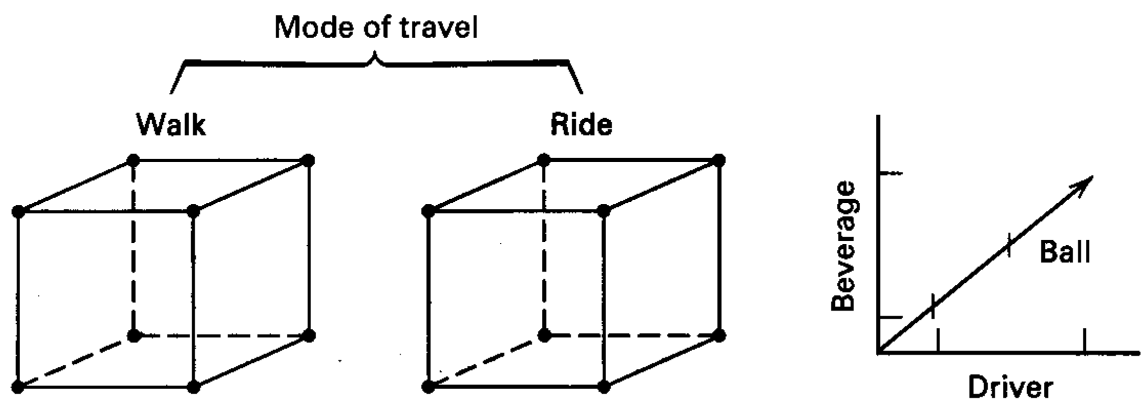


Figure 1-7 A four-factor factorial experiment involving type of driver, type of ball, type of beverage, and mode of travel.

A Fractional Factorial

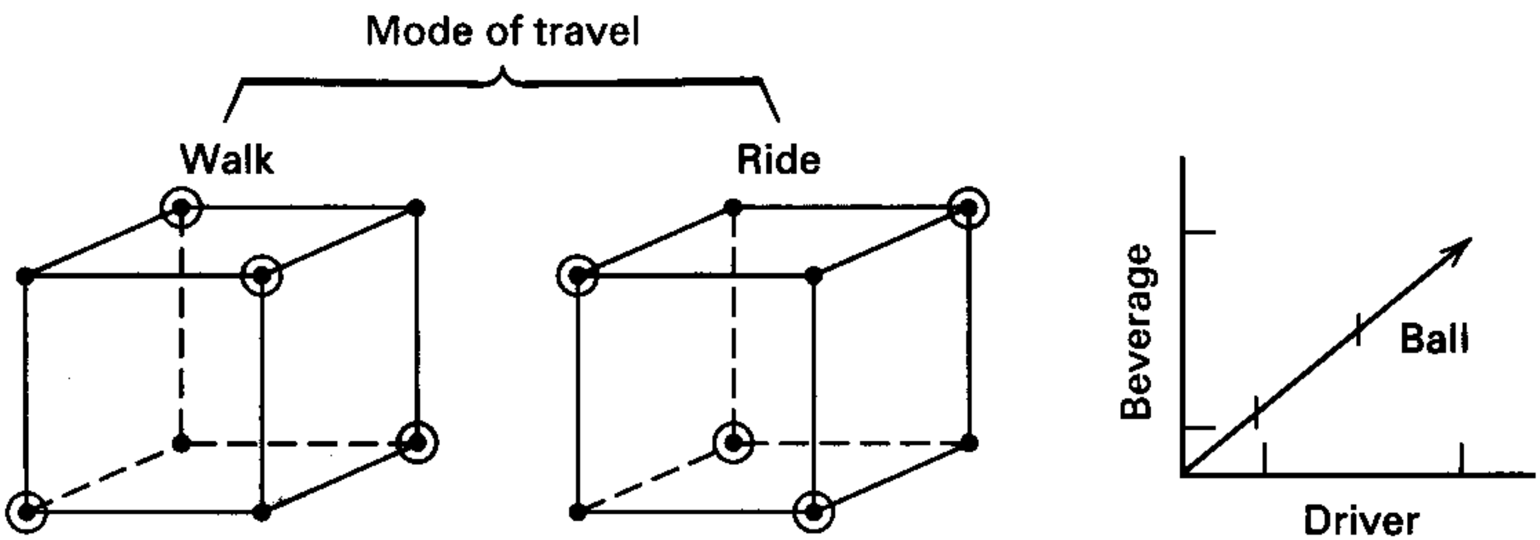


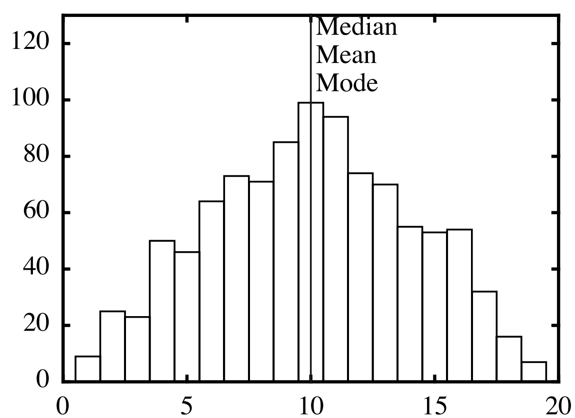
Figure 1-8 A four-factor fractional factorial experiment involving type of driver, type of ball, type of beverage, and mode of travel.

Statistical Rigour

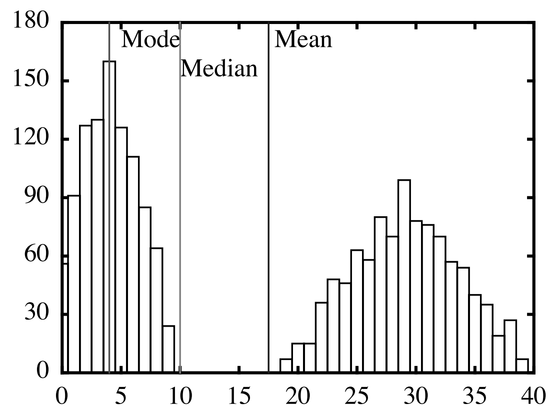
- Understanding common statistical methods is invaluable in being able to represent results coherently and accurately.
- When measuring a system that does not have fixed behaviour:
  - Perform multiple measurements (replicates).
  - Statistical error arises from variation that is uncontrolled; it is generally unavoidable.

Sample Distributions

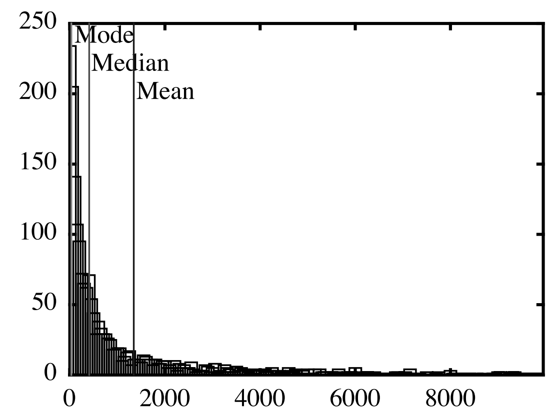
- The relationship between the measurements of centrality (mean, median, and mode) give hints about the distribution of the data collected.



A: Normal Distribution



B: Bimodal Distribution



C: Exponential Distribution

Expressing Variation

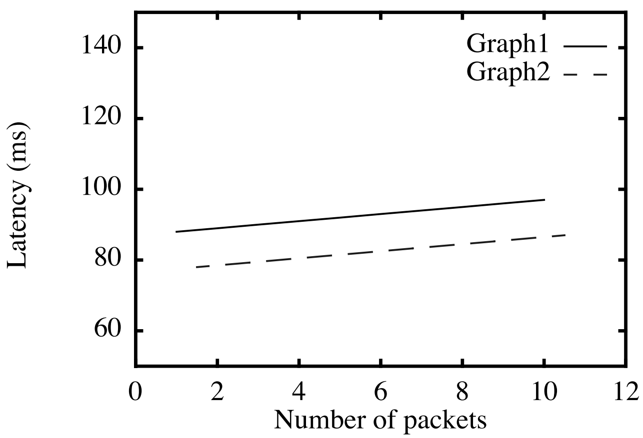
- Measures of centrality are not sufficient to completely describe a data set.
- It is often helpful to include a measure of the variance of the data.
  - A small variance implies that the mean is a good representative of the data, whereas a large variance implies that it is a poor one.
- The most commonly used measure of variance is the standard deviation.

Margin of Error

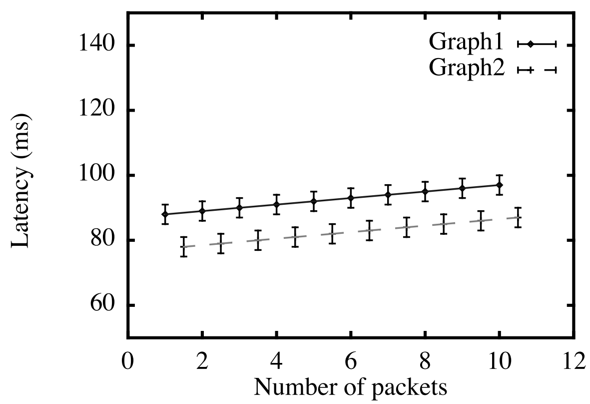
- Another metric for analyzing the usefulness of the mean is the margin of error.
  - The margin of error expresses a range of values about the mean in which there is a high level of confidence that the true value falls.

Graphing and Error Margins

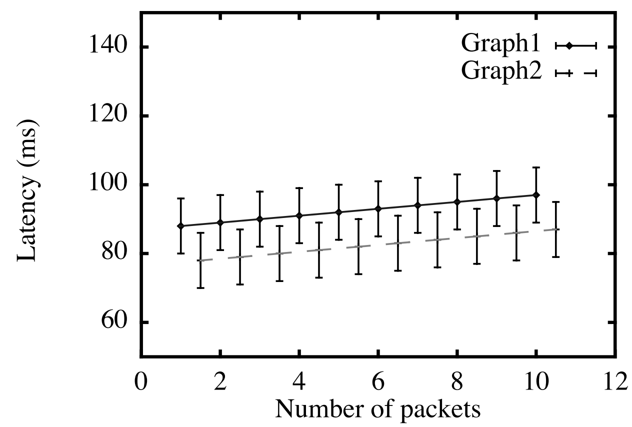
- The value of the error margins depict the results in completely different ways.†



A: Latency Improvement without error margins



B: Latency Improvement with small error margins



C: Latency Improvement with large error margins

† C. Small, N. Ghosh, H. Saleeb, M. Selzter, and K. Smith, “Does Systems Research Measure Up?” Harvard Computer Science Group, Technical Report TR-16-97.

Probability Distributions & Testing

- Plotting a histogram of the values in a sampled data set is easy way to get an idea of what type of distribution the data follows.
- The X^2 test can be used to determine if sampled data follows a specific distribution.
  - X^2 can be used to obtain a p-value from a family of X^2 distributions; the larger the p-value, the higher the probability that the measured distribution matches the candidate distribution.

Basic Statistical Concepts

- Hypothesis testing: A statement either about the parameters of a probability distribution or the parameters of a model.

  $H_0: \mu_1 = \mu_2$ (null hypothesis)

  $H_1: \mu_1 \neq \mu_2$ (alternative hypothesis)

- If the null hypothesis is rejected when it is true, a type I error has occurred.
- If the null hypothesis is not rejected when it is false, a type II error has been made.

Analysis of Variance (ANOVA)

- Analysis of the fixed effects model.
  - Estimation of model parameters.
- Model adequacy checking.
  - The normality assumption.
  - Residuals. Plots in time, versus fitted values, versus other variables.
- Practical interpretation of results.

Response Surface Methodology Framework

- Factor screening.
- Finding the region of the optimum.
- Modelling and optimization of the response.

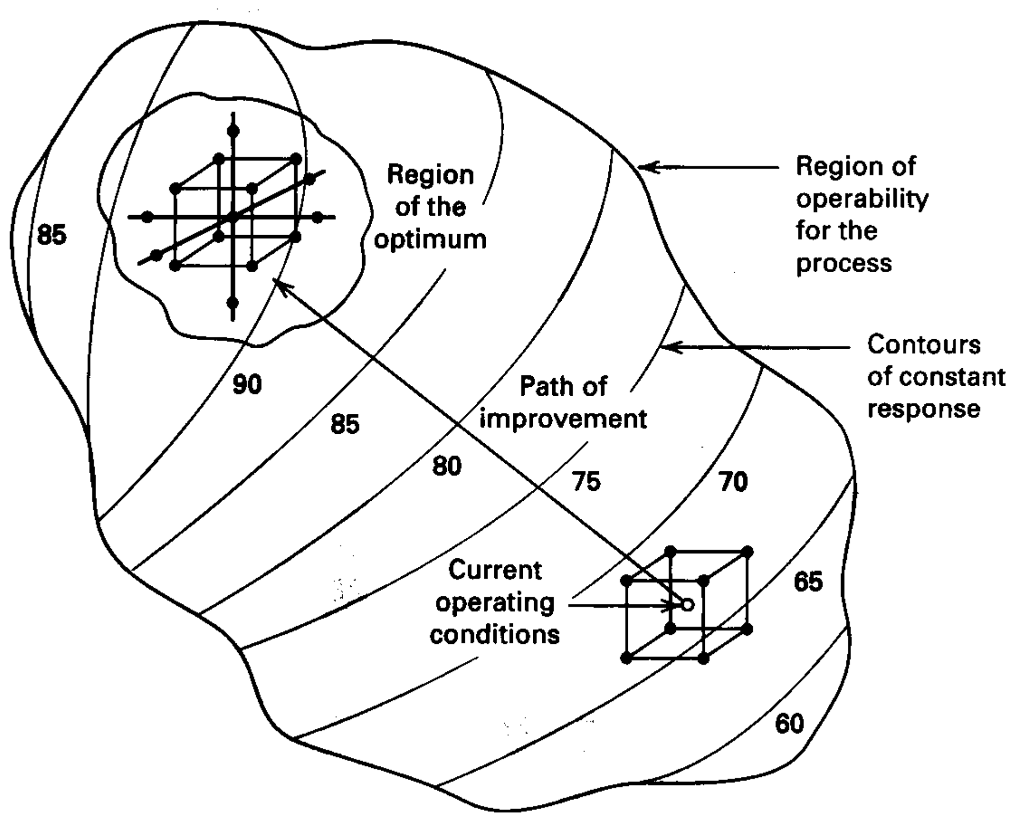


Figure 11-3 The sequential nature of RSM.

Other Aspects of RSM

- Robust parameter design and process robustness studies.
  - Find levels of controllable variables that optimize mean response and minimize variability in the response transmitted from "noise" variables.
  - Original approaches due to Taguchi (1980s).
  - Modern approach based on RSM.

Summary

- There is much known about designing and analyzing experiments!
  - Follow good practices, to improve repeatability and reproducibility of your experiments.