CS 315: Algorithm Design and Analysis

Spring 2013

Homework 2

Assigned: Jan. 29, 2013 Due: Feb. 7, 2013

- 1. (10') Textbook, Exercise 2.4
- 2. (24) Solve the following recurrence relations and give a Θ estimate for each solution. Show the process.
 - (a) T(n) = 3T(n/4) + 1
 - (b) $T(n) = 5T(n/4) + n^{1.5}$
 - (c) $T(n) = 2T(n/2) + n^2$
 - (d) $T(n) = 27T(n/3) + n^3$
 - (e) T(n) = T(n-1) + n
 - (f) T(n) = 2T(n-1) + 4
- 3. (30') Using the divide-and-conquer strategy, design and implement (in C or C++) an algorithm that finds the dominant element of an array of positive integers and returns -1 if it does not have a dominant element.

An element x is the dominant element of an array if more than half of the array's elements are equal to x. For example, 5 is the dominant element of the array with numbers 1, 5, 5, 6, 5, 7, 5, and there is no dominant element in the array with numbers 1, 5, 5, 6, 7, 5.

Your algorithm should run in $O(n \log n)$ time. You are not allowed to sort the array. Assume that elements in the array can be compared for equality, but not for less than, greater than, and the like.

The hint for the algorithm is the fact that if x is the dominant element of A, then x must be either the dominant element of first half of A, or the dominant element of the second half of A, or both. In another word, if y is the dominant element of the first half of A and z is the dominant element of the second half of A, then only y or z can be the dominant element of A.

Here is one way to organize your program. The algorithm implemented will be int find_dominant(int *A, int p, int r), which returns the value of the dominant element in array A[p..r] and returns -1 if no dominant element is found. In the main program, you can declare an array X of size n greater than 10 and initialize it with positive integers. Then it will call find_dominant(X, 0, n-1) and print out the result. Make sure that the complexity of your algorithm is $O(n \log n)$.

Email the program to the TA/grader at weihua.liu@uky.edu and cc to the instructor (fei@cs.uky.edu), with subject "CS315 HW2".

Note that we have changed the submission method. You should submit your program through the CS Web Portal at https://www.cs.uky.edu/csportal